

Supersymmetry Breaking Cascade Flow

from $\mathcal{N} = 2$ to adjoint QCD

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Costas $\cap$ Eric

1978 start graduate school in Princeton

– graduate college, shared an office, working on QCD

1984 Costas and Pascale's wedding in Lille (picture next page)

1995 Eric's engagement party in Santa Barbara

– both working on supersymmetry breaking

1999 celebrate Costas and Pascale fifteenth wedding anniversary

– we collaborate on non-commutative QFT with Boris Pioline

2000's both working on holographic interfaces and defects

2013 we *finally* co-author one paper with John Estes and Darya Krym

"M-theory solutions invariant under $D(2, 1; \gamma) \times D(2, 1; \gamma)$ "

2024 both *retire* !

Costas $\cap$ Eric



Motivation

Seiberg-Witten solution for $\mathcal{N} = 2$ super Yang-Mills

- ★ provides exact low energy effective action and BPS spectrum

Softly breaking supersymmetry

- ★ exploit the enhanced control provided by the SW solution

Earlier investigations into softly breaking $\mathcal{N} = 2$

- ★ confinement via magnetic monopole condensation [Seiberg, Witten 1994]
[Alvarez-Gaumé, Distler, Kounnas, Marino, 1996; Luty, Rattazzi 1999; Edelstein, Fuertes, Mas, Guilarte, 2000]
- ★ embedding $SU(2)$ adjoint QCD into $\mathcal{N} = 2$ [Cordova, Dumitrescu 2018]

This talk

Pure $\mathcal{N} = 2$ super-Yang-Mills with gauge group $SU(N)$

- ★ gauge multiplet $(\phi, \lambda^1, \lambda^2, v_\mu)$ in adjoint representation

Add mass term $M^2 \text{tr}(\phi^\dagger \phi)$ for gauge scalars ϕ

- ★ softly breaks all supersymmetries
- ★ preserves all other symmetries and 't Hooft anomalies
- ★ ϕ decouples as $M \rightarrow \infty$ to adjoint QCD $(\lambda^1, \lambda^2, v_\mu)$ with two flavors

Phase structure along the flow $0 < M < \infty$?

Proposal: a magnetic dual Abelian Higgs model

⇒ Cascade of phase transitions through partial Coulomb/Higgs phases

with Thomas Dumitrescu, Efrat Gerchkovitz and Emily Nardoni

2012.11843; 2208.11502; 2408.xxxxx

$\mathcal{N} = 2$ super Yang-Mills with gauge group $SU(N)$

Coulomb branch vacua $[\phi^\dagger, \phi] = 0$ have unbroken $\mathcal{N} = 2$

$$SU(N) \longrightarrow U(1)^{N-1}$$

Low energy $\mathcal{N} = 2$ gauge $U(1)_k$ multiplets for $k = 1, \dots, N-1$

★ in terms of $\mathcal{N} = 1$ chiral superfields A_k and gauge field strength W_k

$$\mathcal{L}_{\text{SW}} = \frac{\text{Im}}{4\pi} \sum_{k=1}^{N-1} \int d^4\theta A_{Dk} \bar{A}_k + \frac{\text{Im}}{8\pi} \sum_{k,\ell=1}^{n-1} \int d^2\theta \tau_{k\ell} W_k W_\ell$$

★ specified by a locally holomorphic pre-potential $\mathcal{F}(A_1, \dots, A_{N-1})$

$$A_{Dk} = \frac{\partial \mathcal{F}}{\partial A_k} \quad \tau_{k\ell} = \frac{\partial^2 \mathcal{F}}{\partial A_k \partial A_\ell}$$

★ subject to $\text{Im } \tau > 0$ for positive kinetic term of $U(1)_k$ gauge fields

The Seiberg-Witten solution

The Seiberg-Witten solution

- ★ determines the pre-potential $\mathcal{F}$ on the Coulomb branch
- ★ as a function of the vevs $\langle A_k \rangle = a_k$ and $\langle A_{Dk} \rangle = a_{Dk}$

SW curve $\mathcal{C}$ and differential λ for $SU(N)$

[Klemm, Lerche, Yankielowicz, Theisen; Argyres, Faraggi 1994]

- ★ in terms of a degree N polynomial $C(x)$

$$\mathcal{C} = \left\{ (x, y), y^2 = C(x)^2 - \Lambda^{2N} \right\} \quad \lambda = \frac{x C' dx}{2\pi i y}$$

- ★ vevs a_k, a_{Dk} in terms of periods over canonical homology basis

$$a_k = \oint_{\mathfrak{A}_k} \lambda \quad a_{Dk} = \oint_{\mathfrak{B}_k} \lambda$$

- ★ masses of BPS dyons with electric/magnetic charges $\mathbf{q}, \mathbf{m} \in \mathbb{Z}^{N-1}$

$$M_{\text{BPS}} = |Z| \quad Z = \sum_{k=1}^{N-1} (q_k a_k + m_k a_{Dk})$$

Multi-monopole points

- **Near a multi-monopole point** [Douglas, Shenker 1995]
 - ★ $N - 1$ *mutually local* magnetic monopoles become massless
 - ★ magnetic periods $a_{Dk} \rightarrow 0$ so that $M_{\text{BPS}}(\mathbf{0}, \mathbf{m}) \rightarrow 0$
 - ★ electric periods $a_k \not\rightarrow 0$ so that $M_{\text{BPS}}(\mathbf{q}, \mathbf{0})$ remains finite
- **Massless states produce singularities in τ as $a_D \rightarrow 0$**
$$-2\pi i \tau_{k\ell} \approx \delta_{k\ell} \ln \frac{\Lambda}{a_{Dk}}$$
 - ★ the SW low energy effective Lagrangian breaks down because it integrated out light magnetic monopoles
- **Viable low energy effective theory**
 - ★ requires keeping the fields of massless states

Effective Abelian Higgs model

“Integrate in” magnetic monopole fields below RG scale μ

- ★ hyper-multiplets $\mathcal{H}_k$ for gauge group $U(1)_k$ for $k = 1, \dots, N - 1$
- ★ in terms of $\mathcal{N} = 1$ chiral superfields $\mathcal{H}_k^\pm$

Effective Lagrangian including magnetic monopole fields

- ★ dictated by $\mathcal{N} = 2$ supersymmetry

$$\mathcal{L}_{\text{SW}}^{\text{eff}} \supset \bar{\mathcal{H}}_k^\pm e^{\mp 2V_k} \mathcal{H}_k^\pm, \quad \bar{A}_k A_{Dk}, \quad A_{Dk} \mathcal{H}_k^+ \mathcal{H}_k^-, \quad \tau_{Dk\ell}^{\text{eff}} W_k W_\ell$$

- ★ Gauge couplings $\tau_{k\ell}$ are now free of singularities as $a_{Dk} \rightarrow 0$

$$-2\pi i \tau_{k\ell}^{\text{eff}} = \delta_{k\ell} \ln \frac{\Lambda}{\mu} + \ln L_{k\ell} + \mathcal{O}(a_D)$$

- ★ where $L_{k\ell}$ is given by [ED, Dumitrescu, Gerchkovitz, Nardoni 2020]

$$L_{kk} = 16N s_k^3 \quad L_{k \neq \ell} = \frac{1 - c_{k+\ell}}{1 - c_{k-\ell}} \quad \begin{cases} c_k = \cos(k\pi/N) \\ s_k = \sin(k\pi/N) \end{cases}$$

Soft supersymmetry breaking operator

The operator $\mathcal{T} = M^2 \text{tr}(\phi^\dagger \phi)$ in the $\mathcal{N} = 2$ theory

- ★ dimension protected since component of $\mathcal{N} = 2$ stress tensor multiplet
- ★ IR behavior of $\mathcal{T}$ governed by SW theory
- ★ flows towards the Kähler potential

$$\mathcal{T} \rightarrow \frac{i}{4\pi} \sum_{k=1}^{N-1} \left(\bar{a}_{Dk} a_k - \bar{a}_k a_{Dk} \right)$$

- ★ with magnetic monopole hyper-multiplet fields $\mathcal{H}_k^\pm$ integrated in

$$\mathcal{T} \rightarrow \frac{i}{4\pi} \sum_{k=1}^{N-1} \left(\bar{a}_{Dk}^{\text{eff}} a_k^{\text{eff}} - \bar{a}_k^{\text{eff}} a_{Dk}^{\text{eff}} + 2\pi i \bar{h}_k h_k \right)$$

Flow from $\mathcal{N} = 2$ to adjoint QCD by the operator $\mathcal{T}$

- ★ adding $\mathcal{N} = 2$ to the Lagrangian breaks susy completely
- ★ in practice: ignore back-reaction of $\mathcal{T}$ on RG flow

The Abelian Higgs model with susy breaking

The operator $M^2\mathcal{T}$ drives vacuum towards minimum of $\mathcal{T}$

- ★ at the $\mathbb{Z}_{2N}$ symmetric point [ED, Dumitrescu, Nardoni 2022]
- ★ use self-consistent approximation for a_k linear in $a_{D\ell}$
- ★ treat higher order terms perturbatively [ED, Phong 1997]

Assembling effective potential for the Abelian Higgs model $+\mathcal{T}$

$$\begin{aligned}
 M^4 V_{\text{red}} = & \sum_{k=1}^{N-1} \left(\frac{M^2 N \Lambda}{\pi^2} s_k \text{Im}(a_{Dk}) + \left\{ 2|a_{Dk}|^2 - \frac{1}{2}M^2 \right\} \bar{h}_k h_k \right) \\
 & + \sum_{k,\ell=1}^{N-1} \left(M^2 t_{k\ell} a_{Dk} \bar{a}_{D\ell} + (t^{-1})_{k\ell} \left\{ (\bar{h}_k h_\ell)(\bar{h}_\ell h_k) - \frac{1}{2}(\bar{h}_k h_k)(\bar{h}_\ell h_\ell) \right\} \right)
 \end{aligned}$$

Proposal: Abelian Higgs model is dual to flow from $\mathcal{N} = 2$ to adjoint QCD

- ★ for small M back-reaction of $\mathcal{T}$ on flow can be ignored
- ★ for larger M we present evidence for a coherent picture

Vacuum alignment

Minima of $\mathcal{V}$ occur at $\text{Re}(a_{Dk}) = 0$

The Higgs fields h_k align perfectly as $SU(2)_R$ doublets

★ minimize $\mathcal{V}$ for given values of $h_k^\dagger h_k$;

$$\mathcal{V}\big|_{\bar{h}_k h_k} = \sum_{k,\ell} (t^{-1})_{k\ell} \mathbf{v}_k \cdot \mathbf{v}_\ell \quad \mathbf{v}_k = h_k^\dagger \boldsymbol{\sigma} h_k \quad h_k = \begin{pmatrix} h_k^1 \\ h_k^2 \end{pmatrix}$$

★ ground state is ferromagnetic under the mild assumption

$$(t^{-1})_{k\ell} < 0 \quad k \neq \ell$$

★ holds automatically for sufficiently small μ/Λ

$SU(2)_R$ is spontaneously broken as soon as any $h_k \neq 0$

★ producing two Goldstone bosons $SU(2)_R \rightarrow U(1)_R$ i.e. CP^1 -phase

★ matches expected chiral symmetry breaking in adjoint QCD

Organization of semi-classical analysis

Reduced field equations for Higgs VEVs

$$h_k \left(M^2 - 4|a_{Dk}|^2 - 2 \sum_{\ell=1}^{N-1} (t^{-1})_{k\ell} |h_\ell|^2 \right) = 0$$

Solutions to Higgs eqs organized by partitions of $k \in \{1, \dots, N-1\}$

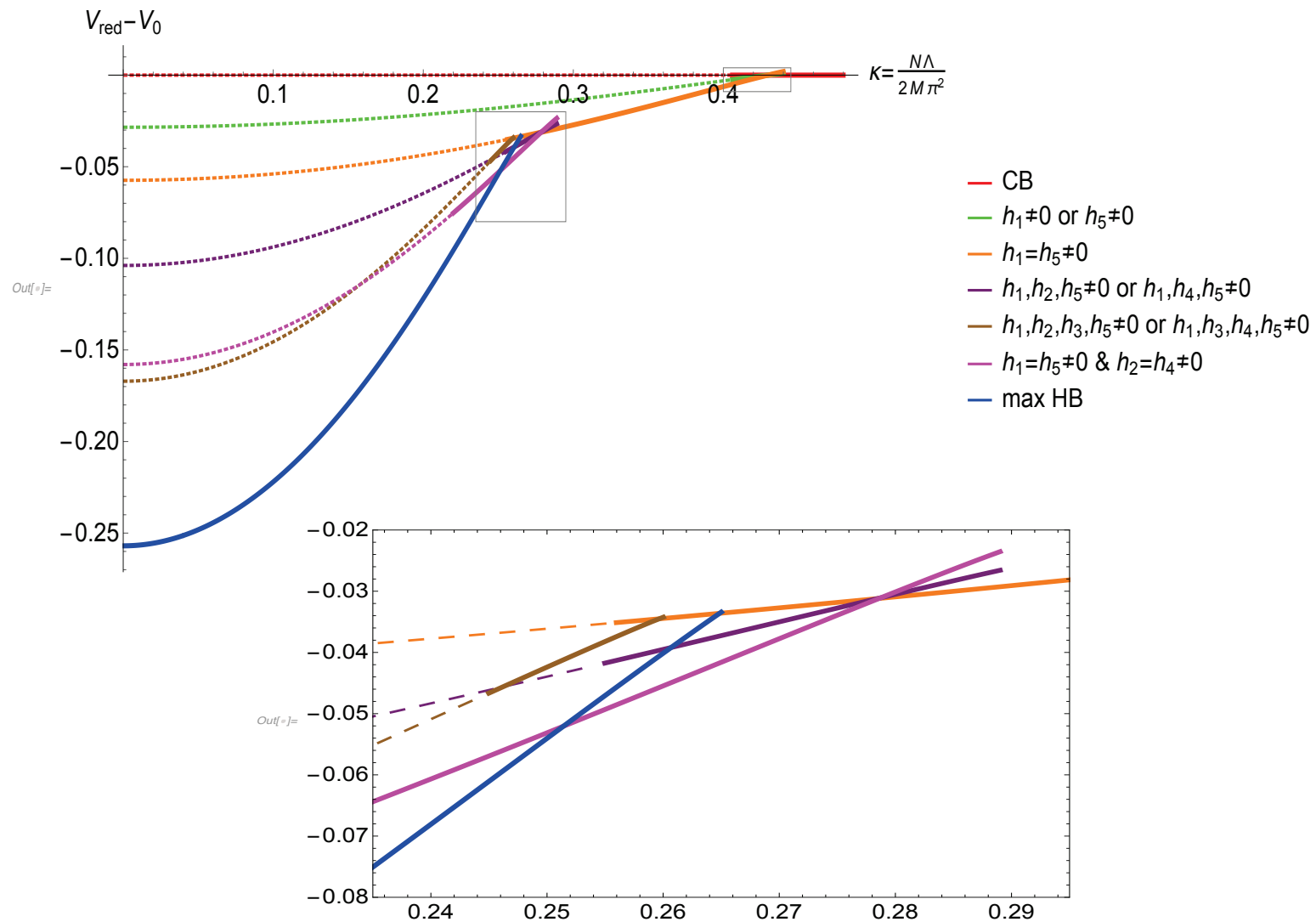
$$h_k = 0 \quad \text{versus} \quad M^2 - 4|a_{Dk}|^2 - 2 \sum_{\ell=1}^{N-1} (t^{-1})_{k\ell} |h_\ell|^2 = 0$$

- ★ Solve for h_k within a given partition in terms of a_D
- ★ Results in coupled cubics in a_{Dk} for $h_k \neq 0$

Steps in semi-classical analysis

- ★ Existence of solutions for given $N, M/\Lambda$
- ★ Local stability of solutions: positive Hessian
- ★ Global stability of solutions: global minimum of $\mathcal{V}$ for given $N, M/\Lambda$

The example of SU(6) gauge group



Summary

Magnetic Abelian Higgs model dual for flow from $\mathcal{N} = 2$ to adjoint QCD

- ★ Magnetic monopole fields have been “integrated in”
- ★ Soft susy breaking operator flows to Kähler potential of SW theory

Semi-classical Abelian Higgs model matches $SU(N)$ theory

- ★ Small M : Coulomb branch
- ★ Large M : monopole condensation
 - matches confinement in adjoint QCD

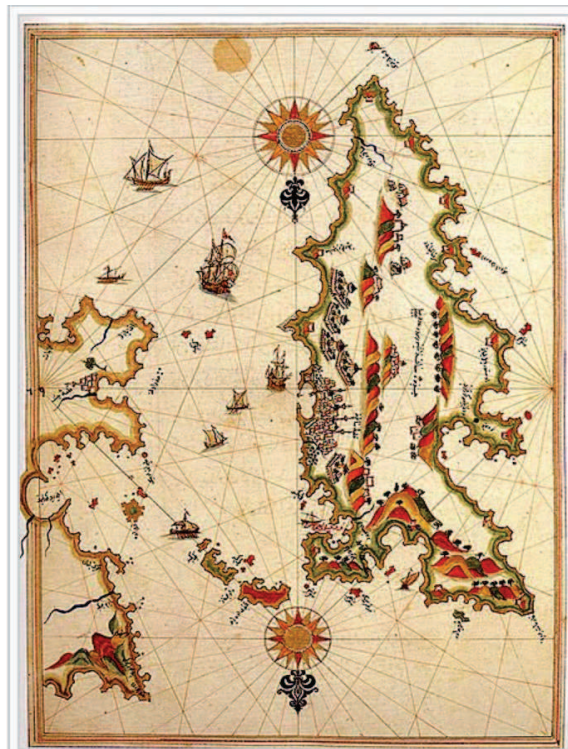
Semi-classical analysis of Abelian Higgs model predicts

- ★ Cascade of phases transitions
 - through mixed Coulomb - Higgs phases

Happy retirement Costas !



... boating on the Seine ...



16th-century detailed map of Chios by [Piri Reis](#)

... on Chios with Pascale ...



... diving for octopus ...